# Recurrent Strings in a OL Language (数理情報科学の基礎理論と応用)

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# Recurrent Strings in a 0L Language

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## 0. *Introduction*

In the original papers of Lindenmayer (3,4), his rewriting system explains the growth of filamentous organisms. But his idea of parallel rewriting has supplied an abundant realm of research to the formal language theory (1,5,6,7). In the context of the formal language theory, an L scheme defines a mapping on the set of all strings over a given finite alphabet. A string is changed into some strings by the L scheme, and these strings are in turn changed into some other strings, and so on. For a string x, there are many descendants of x produced by the mapping of the given L scheme. Among these descendent strings some might go back to the original string x after several operations of the mapping. If every descendent string has a path which goes back to the original string x, we think that the string x has a kind of stability. We call such a string 'recurrent' with respect to the L scheme. Walker and Herman defined an adult string (2,8), which is entirely mapped onto itself. In other words, an adult string is not changed under the L scheme. Obviously, an adult string is a special case of our recurrent string, and our definition is a natural extension of that of Walker's.

From the biological point of view, the recurrentness corresponds to some sort of maturity. Matured organisms seem to make no essential change. According to our definition a recurrent organism can always come back to itself even if it changes into some other one. We believe that the investigation of the recurrentness will shed new lights on the theory of L scheme and L system. That is, new possibility of treating matured organisms will be brought into the framework of L system theory.

In this paper, we give the definitions of a recurrent string and closed strongly connected set with respect to a 0L scheme S. Then we prove a factorization theorem of the recurrent string and a construction theorem of the closed strongly connected set. In the last section we consider some properties of recurrent strings in a 0L language.

## 1. *Preliminaries*

First, we give the definitions of 0L scheme and 0L system.

Definition 1.1. A 0L scheme S is a pair $S=\langle\Sigma,P\rangle$ where $\Sigma$ is a finite alphabet, and P is a finite subset of $\Sigma\times\Sigma^*$(called rewriting rule) such that for any $a\in\Sigma$ there exists at least one $x\in\Sigma^*$ and $(a,x)\in P$. □

We write $a\to x\in P$ or $x\in P(a)$ instead of $(a,x)\in P$. A 0L scheme $S=\langle\Sigma,P\rangle$ defines a relation $\underset{S}{\Longrightarrow}$ over $\Sigma^*$ as follows.

Definition 1.2. Let $S=\langle\Sigma,P\rangle$ be a 0L scheme. For $x,y\in\Sigma^*$, we write $x\underset{S}{\Longrightarrow}y$ if and only if $x=x_1x_2\ldots x_n$, $x_i\in\Sigma$, $y=y_1y_2\ldots y_n$, $y_i\in\Sigma^*$ and $x_i\to y_i\in P$ for $i=1,2,\ldots,n$. □

The reflexive and transitive closure of $\underset{S}{\Longrightarrow}$ is denoted by $\overset{*}{\underset{S}{\Longrightarrow}}$, and n times operation of $\underset{S}{\Longrightarrow}$ by $\overset{n}{\underset{S}{\Longrightarrow}}$. We write $\Longrightarrow(\overset{*}{\Longrightarrow}, \overset{n}{\Longrightarrow})$ instead of $\underset{S}{\Longrightarrow}(\overset{*}{\underset{S}{\Longrightarrow}}, \overset{n}{\underset{S}{\Longrightarrow}})$ when S is understood. The relation $\overset{*}{\Longrightarrow}$ can be regarded as a relation over $2^{\Sigma^*}$. Sometimes we use the notation $\overset{+}{\Longrightarrow}$ which means $\overset{n}{\Longrightarrow}$ for some $n\geq 1$. In L system theory $\overset{*}{\Longrightarrow}$ is usually called a derivation. Note that for every $x\in\Sigma^*$, $x\overset{0}{\Longrightarrow}x$, and for any nonnegative integer n, $\lambda\overset{n}{\Longrightarrow}\lambda$.

Definition 1.3. i) A 0L system G is a triple $G=\langle\Sigma,P,\omega\rangle$ where $\langle\Sigma,P\rangle$ is a 0L scheme and $\omega$ is in $\Sigma^*$ called an axiom.

ii) A 0L language L(G) is given by $L(G)=\{x \mid x\in\Sigma^* \text{ and } \omega\overset{*}{\Longrightarrow}x\}$. □

Now we give an illustrative example of 0L scheme which will be used in the sequel.

<u>Example 1.1.</u> Let $S=\langle\Sigma,P\rangle$ be a 0L scheme, where $\Sigma=\{a,b,c,d,e\}$ and $P=\{a\to ae, a\to b, b\to a, c\to ab, c\to ec, d\to e, d\to c, e\to\lambda, e\to e\}$. If we consider a 0L system $G=\langle\Sigma,P,a\rangle$, then some of the derivations are $a\Longrightarrow ae\Longrightarrow aee\Longrightarrow b$, $a\Longrightarrow b\Longrightarrow a\Longrightarrow ae$ and $a\Longrightarrow ae\Longrightarrow be$. It is easily seen that $L(G)=(a\cup b)e^*$. □

## 2. *Definitions and Lemmas*

In this section we give the definitions of a recurrent string and a closed strongly connected set. We establish some basic results.

<u>Definition 2.1.</u> Let $S=\langle\Sigma,P\rangle$ be a 0L scheme. $x\in\Sigma^*$ is said to be recurrent with respect to S if for any $z\in\Sigma^*$ such that $x\overset{*}{\Longrightarrow}z$, we have $z\overset{*}{\Longrightarrow}x$. □

<u>Definition 2.2.</u> Let $S=\langle\Sigma,P\rangle$ be a 0L scheme and A be a subset of $\Sigma^*$.

i) A is said to be closed with respect to S if for any $x\in A$ and $y\in\Sigma^*$ such that $x\overset{*}{\Longrightarrow}y$ we have $y\in A$.

ii) A is said to be strongly connected with respect to S if for any $x,y\in A$ we have $x\overset{*}{\Longrightarrow}y$. □

<u>Proposition 2.1.</u> With respect to a 0L scheme $S=\langle\Sigma,P\rangle$, $x\in\Sigma^*$ is recurrent if and only if $x\in A$ where A is a closed strongly connected subset of $\Sigma^*$.

<u>Proof.</u> If part: For any $z\in\Sigma^*$ such that $x\overset{*}{\Longrightarrow}z$, we have $z\in A$ because A is closed. As A is also strongly connected, we have $z\overset{*}{\Longrightarrow}x$, which means that x is recurrent.

Only if part: Let $A=L(G)$ where $G=\langle\Sigma,P,x\rangle$. Then A is closed by the definition of L(G). For any $y,z\in A$, there exist derivations

$x \stackrel{*}{\Longrightarrow} y$, $x \stackrel{*}{\Longrightarrow} z$ and $y \stackrel{*}{\Longrightarrow} x$ the last one due to the recurrentness of x. So we have $y \stackrel{*}{\Longrightarrow} z$, and A is strongly connected. □

<u>Example 2.1.</u> Consider the 0L scheme $S=<\Sigma,P>$ in Example 1.1. Then a is recurrent with respect to S. $(a \cup b)e^*$ is closed strongly connected with respect to S. □

If a string $x \neq \lambda$ has a derivation $x \stackrel{+}{\Longrightarrow} \lambda$, then x cannot be recurrent. So we must pick up the 'mortal' symbols as follows.

<u>Definition 2.3.</u> Let $S=<\Sigma,P>$ be a 0L scheme. The set of vital symbols $\Sigma_v \subset \Sigma$ is given by

$$\Sigma_v = \{a \mid a \in \Sigma \text{ and } a \stackrel{*}{\Longrightarrow} x \text{ implies } x \neq \lambda\}.$$

The set of mortal symbols $\Sigma_m \subset \Sigma$ is given by

$$\Sigma_m = \Sigma - \Sigma_v$$

or

$$\Sigma_m = \{b \mid b \in \Sigma \text{ and there is a derivation } b \stackrel{*}{\Longrightarrow} \lambda\}. \quad \square$$

<u>Definition 2.4.</u> Let $x \in \Sigma^*$. The vitality of x (denoted as $v(x)$) equals the number of vital symbols in x. □

If a symbol b is mortal, then there is a derivation $b \stackrel{k}{\Longrightarrow} \lambda$ such that $k \leq \mathrm{card}\Sigma$ where $\mathrm{card}\Sigma$ denotes the cardinality of $\Sigma$. Therefore $\Sigma_m$ and $\Sigma_v$ are effectively constructed and the vitality of a string is effectively computable.

<u>Lemma 2.2.</u> Let $S=<\Sigma,P>$ be a 0L scheme. For any $x,y \in \Sigma^*$, we have the followings.

1) If $x \stackrel{*}{\Longrightarrow} y$, then $v(x) \leq v(y)$.
2) If x is recurrent and $x \stackrel{*}{\Longrightarrow} y$, then $v(x)=v(y)$.

<u>Proof.</u> Obvious. □

The above Lemma tells us that every rewriting rule for a symbol in a recurrent string must be vitality preserving. This motivates us to define a further classification of $\Sigma_m$ and a subscheme of a given 0L scheme as follows.

Definition 2.5. The set of ever mortal symbols $\Sigma_{mm} \subset \Sigma_m$ is given by

$$\Sigma_{mm} = \{a \mid a \in \Sigma_m \text{ and } a \overset{*}{\Longrightarrow} x \text{ implies } v(x)=0\}. \quad \square$$

We denote the remainder part of $\Sigma_m$, i.e., $\Sigma_m - \Sigma_{mm}$ as $\Sigma_{mv}$. If a is in $\Sigma_{mv}$ then there is a derivation $a \overset{k}{\Longrightarrow} x$ and $v(x) \geq 1$. In this case we can assume $k \leq \text{card}\Sigma$. Hence it is decidable whether or not a given symbol a is in $\Sigma_{mm}$.

Definition 2.6. Let $S=\langle\Sigma,P\rangle$ be a 0L scheme. The vitality preserving scheme of S is a 0L scheme $S'=\langle\Sigma',P'\rangle$ where $\Sigma'=\Sigma_v' \cup \Sigma_{mm}$ and $\Sigma_v'$ is the maximum subset of $\Sigma_v$ satisfying

$$P(\Sigma_v') \subset \Sigma_{mm}^* \Sigma_v' \Sigma_{mm}^*$$

and

$$P' \text{ is the restriction of } P \text{ to } \Sigma' \times \Sigma'^*. \quad \square$$

Note that $\Sigma_m' = \Sigma_{mm}' = \Sigma_{mm}$ and $\Sigma_v' \subset \Sigma_v$. Note that in the vitality preserving scheme $\langle\Sigma',P'\rangle$ $x \overset{*}{\Longrightarrow} y$ implies $v(x)=v(y)$ for any $x,y \in \Sigma'^*$.

Proposition 2.3. Let $S=\langle\Sigma,P\rangle$ and $S'=\langle\Sigma',P'\rangle$ be a 0L scheme and its vitality preserving scheme, respectively. A string is recurrent with respect to S if and only if it is recurrent with respect to S'.

Proof. Let $x \in \Sigma^*$ be recurrent with respect to S. By virtue of Lemma 2.2 x must be in $\Sigma'^*$. Because S' is a subscheme of S, x is also recurrent in S'. If $x \in \Sigma'^*$ is recurrent with respect to S', then it is easy to see that x is recurrent in S. $\square$

Let $S'=\langle\Sigma',P'\rangle$ be the vitality preserving scheme of a 0L scheme $S=\langle\Sigma,P\rangle$. We define vital recurrent symbols in $\Sigma_{vr}'$ as follows.

Definition 2.7. The set of vital recurrent symbols $\Sigma'_{vr}$ satisfies the following condition:

$a \in \Sigma'_{vr} \Longleftrightarrow a \in \Sigma'_{v}$ and for any z such that $a \overset{*}{\Longrightarrow} z$ there exists a derivation $z \overset{*}{\Longrightarrow} x$ where x contains a. □

Note that it is decidable whether or not a given symbol is in $\Sigma'_{vr}$.

Example 2.2. Consider the 0L scheme $S=\langle\Sigma,P\rangle$ in Example 1.1. Then $\Sigma_v=\{a,b,c\}$, $\Sigma_m=\{d,e\}$, and $\Sigma_{mm}=\{e\}$. The vitality preserving scheme is $\langle\{a,b,e\},\{a\to ae, a\to b, b\to a, e\to\lambda, e\to e\}\rangle$. $\Sigma'_{vr}=\{a,b\}$. □

## 3. *Recurrent Strings with Respect to a 0L Scheme*

In this section we consider the characteristics of the recurrent string and the closed strongly connected set with respect to a 0L scheme. In view of Lemma 2.2 recurrent strings will be characterized by the vitality. One can also easily see that the vital symbols in a recurrent string must be vital recurrent.

Let $S=\langle\Sigma,P\rangle$ be a 0L scheme. For each $b \in \Sigma_m$ there is a minimum positive integer $k_b$ such that $b \overset{k_b}{\Longrightarrow} \lambda$. Let $k = \max_{b\in\Sigma_m} k_b$. We can easily see that k is effectively computable.

Lemma 3.1. Let $S=\langle\Sigma,P\rangle$ and $S'=\langle\Sigma',P'\rangle$ be a 0L scheme and its vitality preserving scheme, respectively. Then the following conditions are equivalent:

1) x is recurrent with respect to S and $v(x)=1$.
2) $x=lar$ for some $lr \in \Sigma'^{*}_{m}$ and $a \in \Sigma'_{vr}$ such that $a \overset{*}{\Longrightarrow} x \overset{+}{\Longrightarrow} x$.
3) $a \overset{n}{\Longrightarrow} x$ for some $a \in \Sigma'_{vr}$ and $n \geq k$.

Proof. 1)→2): By Proposition 2.3, x is recurrent with respect to S′ and we can write $x=lar$ for some $lr \in \Sigma'^{*}_{m}$ and $a \in \Sigma'_{v}$. If $lr=\lambda$, then a is in $\Sigma'_{vr}$ and there is a derivation $a \overset{+}{\Longrightarrow} a$ because $a=x$ is recurrent. Assume $lr \neq \lambda$, then there is a derivation $lr \overset{+}{\Longrightarrow} \lambda$.

Thus we can have a derivation $x\overset{+}{\Longrightarrow}y$ such that $a\overset{+}{\Longrightarrow}y$. By the recurrent property of x, we also have a derivation $y\overset{*}{\Longrightarrow}x$. Therefore, $a\overset{+}{\Longrightarrow}y\overset{*}{\Longrightarrow}x\overset{+}{\Longrightarrow}y\overset{*}{\Longrightarrow}x$. This proof guarantees that a is in $\Sigma'_{vr}$.
2)→3): Obvious.
3)→1): Let $x\overset{*}{\Longrightarrow}y$. Because $a\in\Sigma'_{vr}$, there is a derivation $y\overset{*}{\Longrightarrow}l_1ar_1$ where $l_1r_1\in\Sigma_m^*$. By the assumption that $a\overset{n}{\Longrightarrow}x$ and $n\geq k$, we have $l_1ar_1\overset{n}{\Longrightarrow}x$. Thus $x\overset{*}{\Longrightarrow}y\overset{*}{\Longrightarrow}l_1ar_1\overset{n}{\Longrightarrow}x$ for any possible y, and we see that x is recurrent with respect to S´ and hence with respect to S. □

Theorem 3.2. Let $S=<\Sigma,P>$ be a 0L scheme and $x\in\Sigma^*$ where $v(x)=k$ for a nonnegative integer k. Then x is recurrent with respect to S if and only if $x=x_1x_2\ldots x_k$ such that $v(x_i)=1$ and $x_i$ is recurrent with respect to S for $i=1,2,\ldots,k$.

Proof. If part: It is sufficient to show that if x and y are recurrent so is xy. Let $xy\overset{n}{\Longrightarrow}z_1z_2$ such that $x\overset{n}{\Longrightarrow}z_1$ and $y\overset{n}{\Longrightarrow}z_2$. As there are derivations $z_1\overset{m_1}{\Longrightarrow}x$ and $z_2\overset{m_2}{\Longrightarrow}y$ for some positive integers $m_1$ and $m_2$, we have derivations $z_1\overset{p}{\Longrightarrow}x$ and $z_2\overset{p}{\Longrightarrow}y$ where $p=(n+m_2-1)(n+m_1)+m_1=(n+m_1-1)(n+m_2)+m_2$.
Only if part: Let $S'=<\Sigma',P'>$ be the vitality preserving scheme of S. If x is recurrent with respect to S´ such that $v(x)=k$, we can write $x=b_1a_1b_2a_2\ldots b_ka_kb_{k+1}$ where $a_i\in\Sigma'_{vr}$ $(i=i,2,\ldots,k)$ and $b_1b_2\ldots b_{k+1}\in\Sigma_m'^*$. Then there exists a nonnegative integer n such that $b_1b_2\ldots b_{k+1}\overset{n}{\Longrightarrow}\lambda$ and $a_i\overset{n}{\Longrightarrow}l'_ia'_ir'_i$, $l'_ir'_i\in\Sigma_m'^*$ for $i=1,2,\ldots,k$. Let $y=l'_1a'_1r'_1l'_2a'_2r'_2\ldots l'_ka'_kr'_k$. Because $x\overset{*}{\Longrightarrow}y$ and x is recurrent there is a derivation $y\overset{*}{\Longrightarrow}x$. Then x can be written as $x=l_1a_1r_1l_2a_2r_2\ldots l_ka_kr_k$ where $l_ir_i\in\Sigma_m'^*$ such that $l'_ia'_ir'_i\overset{*}{\Longrightarrow}l_ia_ir_i$ in that derivation for $i=1,2,\ldots,k$. Because $l_1r_1l_2r_2\ldots l_kr_k=b_1b_2\ldots b_{k+1}\overset{n}{\Longrightarrow}\lambda$, we have $l_ia_ir_i\overset{n}{\Longrightarrow}l'_ia'_ir'_i$ and $a_i\overset{*}{\Longrightarrow}l_ia_ir_i\overset{+}{\Longrightarrow}l_ia_ir_i$. Hence by Lemma 3.1, $l_ia_ir_i=x_i$ is recurrent for $i=1,2,\ldots,k$. □

Theorem 3.2 says that a recurrent string of vitality k is factorized into k segments each of which is recurrent and contains one vital recurrent symbol.

Next we consider the closed strongly connected set. Let $S=<\Sigma,P>$ be a 0L scheme and A be a closed strongly connected set with respect to S. Then for any $x \in A$, $L(G)=A$ where $G=<\Sigma,P,x>$, that is, a closed strongly connected set is a 0L language. We will show that there is a finite set B for S such that $A \cap B^* \neq \emptyset$ for an arbitrary closed strongly connected set A. In other words $A=L(G)$ where $G=<\Sigma,P,\omega>$ for some $\omega \in B^*$.

Definition 3.1. Let $S=<\Sigma,P>$ be a 0L scheme. A closed strongly connected subset of $\Sigma^*$ is said to be an elementary closed strongly connected set (abbreviated as ECSC) with respect to S, if $v(x)=1$ for any $x \in A$. □

Lemma 3.3. For each ECSC A with respect to a 0L scheme S, there exist an $x \in A$ and an $a \in \Sigma'_{vr}$ such that $a \overset{i}{\Longrightarrow} x$ for some $i \leq k$.

Proof. Let $a \in \Sigma'_{vr}$ be contained in some string $y \in A$. By Lemma 3.1 there is a recurrent string x such that $a \overset{i}{\Longrightarrow} x$ for some $i \leq k$. For sufficiently large integer j we have $y \overset{j}{\Longrightarrow} x$. Hence $x \in A$. □

Lemma 3.4. Let $S=<\Sigma,P>$ and $S'=<\Sigma',P'>$ be a 0L scheme and its vitality preserving scheme, respectively. If $n=\mathrm{card}\Sigma'_{vr}$, then there exist at most n ECSCs with respect to S.

Proof. If not, then there exist an $a \in \Sigma'_{vr}$ and two strings lar and $l'ar'$ ($lr, l'r' \in \Sigma_m'^*$) such that they belong to different ECSCs. Observing the only if part proof of Theorem 3.2, for sufficiently large integer m, we have $l'r' \overset{m}{\Longrightarrow} \lambda$ and $a \overset{m}{\Longrightarrow} lar$. Thus we ahve $l'ar' \overset{m}{\Longrightarrow} lar$. This is a contradiction. □

Definition 3.2. Let A be an ECSC and $x \in A$. x is said to be a base string of A, if there is an $a \in \Sigma'_{vr}$ such that $a \overset{i}{\Longrightarrow} x$ and there are no string $y \in A$ and symbol $b \in \Sigma'_{vr}$ such that $b \overset{j}{\Longrightarrow} y$ and $j<i$. □

That is, x is a minimum step derivable string from an element in $\Sigma'_{vr}$. Lemma 3.3 guarantees that the number of such base strings is finite and all the base strings are effectively constructed.

Definition 3.3. Let $S=\langle\Sigma,P\rangle$ be a 0L scheme and let $A_1, A_2, \ldots, A_n$ be the enumeration of the ECSCs with respect to S. A base set $B\subset\Sigma^*$ is given by $B=\{x_1, x_2, \ldots, x_n\}$ where $x_i$ is a base string of $A_i$ for $i=1,2,\ldots,n$. □

Theorem 3.5. Let $S=\langle\Sigma,P\rangle$ be a 0L scheme and B be a base of S. If $A\subset\Sigma^*$ is closed strongly connected with respect to S, then there exists $\omega\in B^*$ such that $A=L(G)$ where $G=\langle\Sigma,P,\omega\rangle$ is a 0L system.
Proof. By Theorem 3.2 for any $x\in A$ such that $v(x)=k$, x has the factorization $x=x_1x_2\ldots x_k$. From the definition of ECSC, $x_i$ belongs to some ECSC $A_{j_i}$ for $i=1,2,\ldots,k$. Then there is a base string $b_{j_i}$ of $A_{j_i}$ such that $b_{j_i}\in B$ and $b_{j_i}\overset{m_i}{\Longrightarrow}x_i$ for some nonnegative integer $m_i$. By applying the same argument of the if part proof of Theorem 3.2, we have $b_{j_1}b_{j_2}\ldots b_{j_k}\overset{m}{\Longrightarrow}x_1x_2\ldots x_k$ for some nonnegative integer m. Let $\omega=b_{j_1}b_{j_2}\ldots b_{j_k}$ and our proof is completed. □

## 4. *Recurrent strings with Respect to 0L System*

In this section we think about a few problems of the recurrent strings with respect to a 0L system. Let us consider the problem to decide for a given 0L system G whether or not there exist recurrent strings in L(G). For example, L(G) in Example 1.1 consists of recurrent strings only. On the other hand, if we consider $G=\langle\Sigma,P,c\rangle$ where $\Sigma$ and P are those of Example 1.1, some of the strings in L(G) are not recurrent. Now we must consider the symbols which can derive vital recurrent symbols.

Definition 4.1. Let $S=<\Sigma,P>$ be a 0L scheme. We define two sets of symbols which can derive the vital recurrent symbols as follows

$$\Sigma_{vd}=\{a|a\in\Sigma_v \text{ and there is a derivation } a\overset{*}{\Longrightarrow}x \text{ such that } x\in(\Sigma_m\cup\Sigma'_{vr})^+\}$$

$$\Sigma_{md}=\{b|b\in\Sigma_m \text{ and there is a derivation } b\overset{*}{\Longrightarrow}x \text{ such that } x\in(\Sigma_m\cup\Sigma'_{vr})^+ \text{ and } v(x)\geq 1\}. \quad \square$$

Obviously $\Sigma'_{vr}\subset\Sigma_{vd}$. If $a\in\Sigma_{vd}$ $(\in\Sigma_{md})$, then there exists a nonnegative integer $k\leq\text{card}\Sigma$ such that $a\overset{k}{\Longrightarrow}x$ and $x\in(\Sigma_m\cup\Sigma'_{vr})^+$. Therefore it is decidable whether or not a given symbol a is in $\Sigma_{vd}$ $(\Sigma_{md})$.

For $x\in\Sigma^*$ alph(x) denoted the set of symbols appearing in x and for $A\subset\Sigma^*$ $\text{alph}(A)=\bigcup_{x\in A}\text{alph}(x)$.

Theorem 4.1. Let $G=<\Sigma,P,\omega>$ be a 0L system. L(G) contains a recurrent string which is not $\lambda$ if and only if the following condition holds.
In case $v(\omega)\geq 1$; $\text{alph}\omega\subset\Sigma_{vd}\cup\Sigma_m$.
In case $v(\omega)=0$; $\text{alph}\omega\cap\Sigma_{md}\neq\emptyset$.

Proof. If part: Obvious.
Only if part: First assume $v(\omega)\geq 1$. If $\text{alph}\omega\subset\Sigma_{vd}\cup\Sigma_m$ fails to hold, in other words if there exists $a\in\text{alph}\omega\cap(\Sigma_v-\Sigma_{vd})$, then there exists a symbol $b\in\Sigma_v-\Sigma'_{vr}$ in any descendant of a. By the factorization theorem, the vital symbols contained in a recurrent string must be vital recurrent. This is a contradiction. Next assume $v(\omega)=0$. Let x be a recurrent string in L(G). Then some symbol b in $\omega$ must derive a substring $x'$ of x which contains some elements of $\Sigma'_{vr}$. From the definition of $\Sigma_{md}$, b is in $\Sigma_{md}$. $\square$

Note that in case $v(\omega)=0$ there is always the recurrent string $\lambda$ in L(G). From Theorem 4.1 we have the following

Theorem 4.2. Let $G=<\Sigma,P,\omega>$ be a 0L system. It is decidable whether or not there are recurrent strings in $L(G)$. □

Finally we compare the recurrent language with the adult language of a 0L system. The recurrent and adult languages are defined as follows.

Definition 4.2. Let $S=<\Sigma,P>$ be a 0L scheme and $G=<\Sigma,P,\omega>$ be a 0L system where $\Sigma$ and $P$ are those of $S$ and $\omega\in\Sigma^*$.

i) $x$ is said to be adult with respect to $S$ if $x\overset{*}{\Longrightarrow}y$ implies $y=x$.

ii) An adult language of $G$ is defined as $A(G)=\{x|x\in L(G)$ and $x$ is adult with respect to $S\}$.

iii) A recurrent language of $G$ is defined as $R(G)=\{x|x\in L(G)$ and $x$ is recurrent with respect to $S\}$. □

$\mathcal{A}(0L)$ and $\mathcal{R}(0L)$ denote the family of adult languages and the family of recurrent languages for 0L systems, respectively. The following results are known (2).

Lemma 4.3. There exists an algorithm which takes as input any 0L system $G$ and produces as output a 0L system $H=<\Sigma,P,s>$ such that $A(H)=A(G)$, and for each $a\in alph(A(H))$, $P$ contains $a\rightarrow a$ and no other production with $a$ on the left. □

Theorem 4.4. $\mathcal{A}(0L)=\mathcal{F}(CF)$. □

Theorem 4.5. $\mathcal{A}(0L)\subset\mathcal{R}(0L)$.

Proof. Let $H=<\Sigma,P,s>$ be a 0L system and $A(H)$ be an adult language. $P$ may be assumed to have the form of Lemma 4.3. Define $\Delta$ as follows

$$\Delta=\{a|a\in alph(x) \text{ and } x \text{ appears in the derivation } s\overset{*}{\Longrightarrow}A(H)\}.$$

Let $K=<\Delta,P',s>$ where $P'=P|_{\Delta\times\Delta^*}$. Clearly $A(H)=A(K)\subset R(K)$. If $x\in L(K)$ and $x$ is not adult, then there is an adult string $y$ such that $x\overset{+}{\Longrightarrow}y$, since any $a\in\Delta$ has a derivation $a\overset{*}{\Longrightarrow}z$ where $z$ is adult. From the definition of adult string we cannot have $y\overset{*}{\Longrightarrow}x$, thus all the recurrent strings in $L(K)$ are adult. □

Corollary 4.6. $\mathcal{J}(CF) \subset \mathcal{R}(0L)$. □

It is not known whether or not the above inclusion is proper, but we conjecture $\mathcal{J}(CF) = \mathcal{R}(0L)$. The proof of Theorem 4.4 is a straightforward constructive one and in which Lemma 4.3 plays an essential role. Obviously Lemma 4.3 does not hold for the recurrent case. Some other method will be required for the proof of our conjecture.

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